Cosmic String Configuration in the Supersymmetric CSKR Theory

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Abstract

We study a cosmic string solution of an N=1-supersymmetric version of the Cremmer-Scherk-Kalb-Ramond (CSKR) model coupled to scalars and fermions. The 2-form gauge potential is proposed to couple non-minimally to matter, here described by a chiral scalar superfield. The important outcome is that supersymmetry is kept exact in the core and it may also hold in the exterior region of the string. We contemplate the configurations of the bosonic sector and we check that the solutions saturate the Bogomol'nyi bound. A glimpse on the fermionic zero modes is also given.

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1 Introduction

High-Energy Physics and theories for fundamental interactions strongly rely on the concept of spontaneous symmetry breaking. In Cosmology, the common belief is that, at high temperatures, symmetries that are spontaneously broken today have already been exact in the early stages of the Universe. During the evolution of the Universe, there were various phase transitions, associated with the chain of spontaneous breakdowns of gauge symmetries.

Topological defects such as cosmic strings [1, 2, 3, 4], probably produced during "non-perfect symmetry-breaking transitions" [5] are the object of our concern in this paper. Cosmic strings appear in some Grand Unified Gauge Theories and carry a large energy density [2]. In this way, they may provide a possible origin for the seed density perturbations which became the large scale structure of the Universe observed today [6, 7]. These fluctuations would leave their imprint in the cosmic microwave background radiation (CMBR) that would act as seeds for structure formation and then as builders of the large-scale structures in the Universe[8]. They may also help to explain the most energetic events in the Universe such as high energy cosmic rays[9, 10, 11].

In view of the possibility that Supersymmetry (su.sy) was realized in the early Universe and was broken approximately at the same time as cosmic strings were formed, many recent works investigate cosmic strings by adopting a supersymmetric framework [12, 13].

In the present work, we propose a possible model for supersymmetric cosmic strings. More specifically, we shall be dealing with consider the N=1-version of the Cremmer-Scherk-Kalb-Ramond theory (CSKR). The anti-symmetric Kalb-Ramond tensor field (KR) was first introduced within a string theory context[14]. More generally, p-form gauge fields appear in many supergravity models [15]. In the cosmic string context, the KR-field is an important ingredient to describe a global $U(1)$-string [16, 17, 18, 19], mainly because it brings about a massless scalar that is responsible for a long-range attractive interaction.

To build up the supersymmetric extension of the CSKR model of our paper, three superfields are required; it has a rather simple field content and provides interesting results: the Kalb-Ramond superfield is responsible for the stablility of the potential, the vacuum is supersymmetric, supersymmetry is not broken in the string core, and gauge symmetry is broken without a Fayet-Iliopoulos term. The interesting feature of our model is that su.sy may be kept exact, i. e. we may have a model where a genuinely supersymmetric cosmic string appears, su.sy. being exact in the string core and there is enough freedom to arrange parameters in such a way that sy.sy. may also hold outside the string.

The outline of this paper is as follows: in Section 2, we start by presenting the (CSKR) model in terms of superfields, a component-field description is also carried out and the on-shell version is finally presented. In Section 3, we devote some time to calculate and discuss the gauge-field propagators. Their poles are relevant in order to identify the quanta that intermediate the interactions inside and outside of the string. Problems like gauge and supersymmetry breakings, field equations and vortex configurations, as well the behaviour of the solutions inside and outside the defect, are the matter of Section 4. Section 5 is devoted to the study of the Bogomol′nyi duality conditions. Finally, in Section 6, we draw

our General Conclusions.

2 Setting up a supersymmetric cosmic strings

In this section, we study the $N = 1$-supersymmetric version of the CSKR model with non-minimal coupling to matter, envisaging the study of the formation of a cosmic string.

The ingredient superfields of the model are a chiral scalar supermultiplet, Φ, the gauge superpotential, $\mathcal{V}$, and a chiral spinor-valued superfield, $\mathcal{G}$ [20].

The superspace action is given as follows

$$S = \int d^4x d^2\theta \left\{ -\frac{1}{8}\mathcal{W}^a\mathcal{W}_a + d^2\bar{\theta}\left[-\frac{1}{2}\mathcal{G}^2 + \frac{1}{2}m\mathcal{V}\mathcal{G} + \frac{1}{16}\bar{\Phi}e^{2h\mathcal{V}}\Phi e^{4g\mathcal{G}} \right] \right\}. \tag{2.1}$$

Below, we give the θ-component expressions of the superfield above, where $\mathcal{V}$ is already assumed to be in the Wess-Zumino gauge:

$$\Phi = e^{-i\theta\sigma^\mu\bar{\theta}\partial_\mu}[\phi(x) + \theta^a\chi_a(x) + \theta^2 S(x)]; \tag{2.2}$$

$$\mathcal{V} = \theta\sigma^\mu\bar{\theta}A_\mu(x) + \theta^2\bar{\theta}\bar{\lambda}(x) + \bar{\theta}^2\theta\lambda(x) + \theta\bar{\theta}^2\Delta(x); \tag{2.3}$$

$$\mathcal{G} = \begin{aligned} &-\frac{1}{2}M + \frac{i}{4}\theta\xi_a - \frac{i}{4}\bar{\theta}_{\dot{a}}\bar{\xi}^{\dot{a}} + \frac{1}{2}\theta\sigma^\mu_{a\dot{a}}\bar{\theta}^{\dot{a}}\tilde{G}_\mu + \frac{1}{8}\theta^a\sigma^\mu_{a\dot{a}}\bar{\theta}^2\partial_\mu\bar{\xi}^{\dot{a}} + \\ &-\frac{1}{8}\theta^2\sigma^\mu_{a\dot{a}}\bar{\theta}^{\dot{a}}\partial_\mu\xi^a - \frac{1}{8}\theta^2\bar{\theta}^2\Box M; \end{aligned} \tag{2.4}$$

with the superfield-strenght $\mathcal{W}_a$ written as

$$\mathcal{W}_a = -\frac{1}{4}\bar{D}^2 D^a \mathcal{V}. \tag{2.5}$$

$\tilde{G}_\mu$ is the dual of the 3-form field strength, $G_{\mu\nu k}$, related to the 2-form Kalb-Ramond (KR) field, $B_{\mu\nu}$:

$$G_{\alpha\mu\nu} = \partial_\alpha B_{\mu\nu} + \partial_\mu B_{\nu\alpha} + \partial_\nu B_{\alpha\mu}, \tag{2.6}$$

$$\tilde{G}_\mu = \frac{1}{3!}\epsilon_{\mu\nu\alpha\beta}G^{\nu\alpha\beta}. \tag{2.7}$$

It is worthehile to mention that, though the superfield $\mathcal{V}$ (where A^μ is accommodated) appears explicitly in the action of the model, the $B_{\mu\nu}$-field shows up only through is field-strength, located in the chiral scalar superfield $\mathcal{G}$. As for the χ, λ and ξ, these are the fermionic partners of the scalar matter, the photon and the KR gauge potential, respectively. Our conventions for the supersymmetry (su.sy.) covariant derivatives are given as below:

$$D_a = \partial_a - i\sigma^\mu_{a\dot{a}}\bar{\theta}^{\dot{a}}\partial_\mu,$$
$$\bar{D}_{\dot{a}} = -\partial_{\dot{a}} + i\theta^a\sigma^\mu_{\dot{a}a}\partial_\mu,$$
$$(2.8)$$

where the σ^μ-matrices read as $\sigma^\mu \equiv (\mathbf{1}; \sigma^i)$, the σ^i's being the Pauli matrices.

For a detailed description of the component fields and degrees of freedom displayed in (2.2-2.4), the reader is referred to the paper of ref. [20].

As it can be readily checked, this action is invariant under two independent sets of Abelian gauge transformations, with parameters

$$
\begin{array}{rl}
\phi(x) & \rightarrow \quad \phi'(x) = \phi(x)e^{i\Lambda(x)}, \\
A_\mu(x) & \rightarrow \quad A'_\mu(x) = A_\mu(x) - \partial_\mu\Lambda(x), \\
B_{\mu\nu}(x) & \rightarrow \quad B'_{\mu\nu}(x) = B_{\mu\nu}(x) + \partial_\mu\xi_\nu(x) - \partial_\nu\xi_\mu(x),
\end{array}
\tag{2.9}
$$

where the arbitrary functions Λ and ξ_μ vanish at infinity. The topological term present in (2.1), $m\mathcal{V}\mathcal{G}$, will play a crucial role in the analysis of the breaking of supersymmetry, as we shall find out in next section.

When we write the action in terms of components fields, we obtain the following equations for the auxiliary fields:

$$S^* + \frac{ig}{2}\bar{X}\Gamma_R\Xi + \frac{g^2}{4}\bar{\Xi}\Gamma_R\Xi\phi^* = 0; \tag{2.10}$$

$$\Delta + \frac{1}{2}\left[he^{-2gM}\phi\phi^*\right] - 2mM = 0. \tag{2.11}$$

where X, Ξ and Λ are 4-component spinors so defined that:

$$\Xi \equiv \left(\begin{array}{c} \xi_a(x) \\ \bar{\xi}^{\dot{a}}(x) \end{array} \right), \quad X \equiv \left(\begin{array}{c} \chi_a(x) \\ \bar{\chi}^{\dot{a}}(x) \end{array} \right), \quad \Lambda \equiv \left(\begin{array}{c} \lambda_a(x) \\ \bar{\lambda}^{\dot{a}}(x) \end{array} \right); \tag{2.12}$$

in this 4-dimensional representation, the Dirac matrices are given as below:

$$\Gamma^\mu = \left(\begin{array}{cc} 0 & \sigma^\mu_{ab} \\ \bar{\sigma}^{\mu\dot{a}b} & 0 \end{array} \right), \quad \Gamma_5 \equiv i\Gamma^0\Gamma^1\Gamma^2\Gamma^3. \tag{2.13}$$

Eq.(2.12) is simply a rewriting of the Majorana spinors, Ξ, χ and Λ, in terms of their chiral (L- and R- handed) components.

Using the equation of motion for Δ, we have:

$$U = \frac{1}{2}\left[h|\phi|^2e^{-2gM} - 2mM\right]^2 \geq 0. \tag{2.14}$$

This potential shall be thoroughly discussed in the next section, in connection with the study of the breakdown of the $U(1)$-symmetry supersymmetry in the core and outside the defect. Notice that U is non-negative, as expected by virtue of supersymmetry.

Using (2.10) and (2.11), we next eliminate S and Δ from the component-field action and rewrite the Lagrangian density exclusively in terms of the physical component fields. The Lagrangian takes now the form:

$$\mathcal{L} = \mathcal{L}_B + \mathcal{L}_F + \mathcal{L}_Y - U, \tag{2.15}$$

where the bosonic part, $\mathcal{L}_B$, reads:

$$\begin{aligned}
\mathcal{L}_B = & \nabla_\mu \phi [\nabla^\mu \phi]^* e^{-2gM} + \mathcal{P} \partial_\mu M \partial^\mu M - \tfrac{1}{4} F_{\mu\nu} F^{\mu\nu} \\
& + \tfrac{1}{6} G_{\mu\nu k} G^{\mu\nu k} + m \epsilon^{\mu\nu\alpha\beta} A_\mu \partial_\nu B_{\alpha\beta},
\end{aligned} \tag{2.16}$$

with

$$\mathcal{P} = 1 - g^2 |\phi|^2 e^{-2gM}. \tag{2.17}$$

The kinetic Lagrangian for the fermionic sector is given by

$$\mathcal{L}_F = \frac{i}{2} \bar{\Lambda} \Gamma^\mu \partial_\mu \Lambda + \frac{i}{4} \mathcal{P} \bar{\Xi} \Gamma^\mu \partial_\mu \Xi + \frac{i}{4} \bar{X} \Gamma^\mu \nabla_{\mu 5} X e^{-2gM}, \tag{2.18}$$

whereas the Yukawa Lagrangian is given as below:

$$\begin{aligned}
\mathcal{L}_Y = & i\mathcal{N} \bar{\Lambda} \Gamma_5 \Xi + \left[-h(\phi \bar{\Lambda} \Gamma_R X + \phi^* \bar{\Lambda} \Gamma_L X) - \tfrac{g^2}{4h} \bar{\Xi} \Gamma_5 \Gamma^\mu \mathcal{J}_\mu e^{2gM} \Xi \right. \\
& + -\tfrac{g^2}{2} \partial_\mu M (\bar{X} \Gamma_L \Gamma^\mu \Xi \phi^* + \bar{\Xi} \Gamma_L \Gamma^\mu X \phi) + \tfrac{g}{2} (\Xi \Gamma^\mu \Gamma_R X \nabla_\mu \phi \\
& \left. + \bar{X} \Gamma_L \Gamma^\mu \Xi [\nabla_\mu \phi]^*) \right] e^{-2gM},
\end{aligned} \tag{2.19}$$

where

$$\mathcal{N} = m + gh|\phi|^2 e^{-2gM}. \tag{2.20}$$

and $\Gamma_{L,R}$ are the left- and right-sector projectors, respectively.

The current $\mathcal{J}_\mu$ that appears in $\mathcal{L}_Y$ can be expressed according to:

$$\mathcal{J}_\mu = -\frac{ih}{2} \left(\phi^* \nabla_\mu \phi - \phi [\nabla_\mu \phi]^* \right) e^{-2gM}, \tag{2.21}$$

with

$$\nabla_\mu \phi = \left(\partial_\mu + ih A_\mu + ig \tilde{G}_\mu \right) \phi. \tag{2.22}$$

At last, the covariant derivative with Γ_5-couplings is given by

$$\nabla_{\mu 5} X = \left(\partial_\mu - ih A_\mu \Gamma_5 - ig \tilde{G}_\mu \Gamma_5 \right) X. \tag{2.23}$$

The topological current, namely, the one whose divergencelessness follows without any reference to equations of motion or to symmetries is given by

$$K^\mu = \partial_\nu (\tilde{F}^{\mu\nu} + \tilde{B}^{\mu\nu}). \tag{2.24}$$

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However, due to the absence of magnetic monopoles the first term can be thrown away and we get

$$K^\mu = \partial_\nu \tilde{B}^{\mu\nu}, \tag{2.25}$$

$$\partial_\mu K^\mu \equiv 0. \tag{2.26}$$

The corresponding topological charge is

$$Q \equiv \int K^0 d^3x = \int d^3x \mathcal{B}, \tag{2.27}$$

where $\mathcal{B}$ stands for the magnetic-like field (a scalar) associated to the 2-form potential, which is known to be the divergence of a vector potential:

$$\mathcal{B} = \vec{\nabla}.\vec{b}. \tag{2.28}$$

The model is now completely set up. Next, we shall discuss the spectrum of excitations in order to get relevant information on the interactions mediated by the gauge particles.

3 Propagators and Interaction Range.

The aim of this section is to compute the propagators for the gauge-field excitations. For the sake of generality, we shall suppose that both scalars, ϕ and M, acquire non-vanishing vacuum expectation values, $< \phi >= \eta$ and $< M >= \rho$. Whether or not η and ρ vanishing will be seen when we will study the minimum of the potential and we will discuss the spontaneous breaking of the symmetries (Section 4).

For this purpose, we parametrize ϕ as below:

$$\phi = [\phi(x)' + \eta]e^{i\Sigma(x)}, \tag{3.1}$$

where ϕ' is the quantum fluctuation around the ground state η.

To analyse the propagators, we need to concentrate on the bosonic Lagrangian in terms of the physical fields ϕ', A_μ and $B_{\mu\nu}$. For the sake of reading off these propagators, we refer to the bilinear sector of the bosonic Lagrangian, whose kinetic piece can be cast like below:

$$\mathcal{L}_K = \begin{aligned}[t] &\tfrac{1}{4}F_{\mu\nu}F^{\mu\nu} - (1 - g^2\zeta\eta^2)\tilde{G}_\mu\tilde{G}^\mu + 2(m + gh\zeta\eta^2)A_\mu\tilde{G}^\mu + \\ &-2h\zeta\eta^2\Sigma\partial_\mu A^\mu + h^2\zeta\eta^2 A_\mu A^\mu + \zeta\eta^2\partial_\mu\Sigma\partial^\mu\Sigma, \end{aligned} \tag{3.2}$$

where $\zeta \equiv e^{-2g\rho} = e^{-2g<M>}$.

At first, we will write it in a more convenient form:

$$\mathcal{L} = \frac{1}{2}\sum_{\alpha\beta}\mathcal{A}^\alpha\mathcal{O}_{\alpha\beta}\mathcal{A}^\beta, \tag{3.3}$$

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where $\mathcal{A}_\alpha = (\Sigma, A_\mu, B_{\mu\nu})$ and $\mathcal{O}_{\alpha\beta}$ is the wave operator. We notice that Σ mixes with A^μ. However, if we adopt the t'Hooft R_ξ-gauge, they decouple from each other. So, the $\Sigma - \Sigma$ propagator can be derived independently from the propagators for the $(A^\mu, B^{\mu\nu})$ sector.

In order to invert the wave operator, we have to fix the gauge so as to make the matrix non-singular. This is accomplished by adding the gauge-fixing terms,

$$\mathcal{L}_{A_\mu} = \frac{1}{2\alpha}(\partial_\mu A^\mu + \alpha\zeta h^2 \eta^2 \Sigma)^2; \tag{3.4}$$

$$\mathcal{L}_{B_{\mu\nu}} = \frac{1}{2\beta}(\partial_\mu B^{\mu\nu})^2. \tag{3.5}$$

To read off the gauge-field propagators, we shall use an extension of the spin-projection operator formalism presented in [21, 22]. In this work, we have to add other new operators coming from the Kalb-Ramond terms. The two operators which act on the tensor field are:

$$(P_b^1)_{\mu\nu,\rho\sigma} = \frac{1}{2}(\Theta_{\mu\rho}\Theta_{\nu\sigma} - \Theta_{\mu\sigma}\Theta_{\nu\rho}), \tag{3.6}$$

$$(P_e^1)_{\mu\nu,\rho\sigma} = \frac{1}{2}(\Theta_{\mu\rho}\Omega_{\nu\sigma} - \Theta_{\mu\sigma}\Omega_{\nu\rho} - \Theta_{\nu\rho}\Omega_{\mu\sigma} + \Theta_{\nu\sigma}\Omega_{\mu\rho}), \tag{3.7}$$

where $\Theta_{\mu\nu}$ and $\Omega_{\mu\nu}$ are, respectively, the transverse and longitudinal projection operators, given by:

$$\Theta_{\mu\nu} = \eta_{\mu\nu} - \Omega_{\mu\nu}, \tag{3.8}$$

and

$$\Omega_{\mu\nu} = \frac{\partial_\mu \partial_\nu}{\Box}. \tag{3.9}$$

The other operator coming from the Kalb-Ramond sector, $S_{\mu\gamma k}$, is defined in terms of Levi-Civita tensor as

$$S_{\mu\gamma k} = \epsilon_{\lambda\mu\gamma k}\partial^\lambda. \tag{3.10}$$

In order to find the wave operator's inverse, let us calculate the products of operators for all non-trivial combinations involving the projectors. The relevant multiplication rules are as follows:

$$(P_b^1)_{\alpha\beta,\lambda\xi}S^{\lambda\xi\nu} = 2\epsilon_{\rho\alpha k}{}^{\nu}\partial^{\rho};$$

$$S_{\alpha\lambda\rho}(P_b^1)^{\lambda\rho,\xi\eta} = 2\epsilon_{\rho\alpha}{}^{\xi\eta}\partial^{\rho};$$

$$(P_e^1)_{\alpha\beta,\lambda\xi}S^{\lambda\xi\nu} = 0;$$

$$S_{\alpha\lambda\rho}(P_e^1)^{\lambda\rho,\xi\eta} = 0.$$

(3.11)

The wave operator $\mathcal{O}$ can be split into four sectors, according to:

$$\mathcal{O} = \begin{pmatrix} A & B \\ C & D \end{pmatrix},$$

(3.12)

with

$$A = M_1\Theta_{\mu\nu} + M_2\Omega_{\mu\nu};$$

(3.13)

$$B = -M_3 S_{\mu\nu k};$$

(3.14)

$$C = M_3 S_{\alpha\beta\nu};$$

(3.15)

$$D = M_4(P_b^1)_{\alpha\beta,\nu k} - M_5(P_e^1)_{\alpha\beta,\nu k},$$

(3.16)

where $M_1, ..., M_5$ are quantities that read as the following expressions:

$$M_1 = \tfrac{1}{2}(\Box + 2h^2\zeta\eta^2),$$

$$M_2 = -\tfrac{1}{2}(\tfrac{1}{\alpha}\Box - 2h^2\zeta\eta^2),$$

$$M_3 = \tfrac{1}{2}(m + hg\zeta\eta^2),$$

(3.17)

$$M_4 = \tfrac{1}{2}(1 - g^2\zeta\eta^2)\Box,$$

$$M_5 = \tfrac{1}{8\beta}\Box,$$

ζ was defined in the Lagrangian (3.2).

After lengthy algebraic calculations, we are able to read off the operator $\mathcal{O}^{-1}$.

$$\mathcal{O}^{-1} = \begin{pmatrix} X & Y \\ Z & W \end{pmatrix},$$

(3.18)

where the quantities X, Y, Z and W are:

$$X = (A - BD^{-1}C)^{-1},$$
$$Z = -D^{-1}CX,$$
$$W = (D - CA^{-1}B)^{-1}, \quad (3.19)$$
$$Y = -A^{-1}BW,$$

or, explicitly, in terms of matrix elements,

$$<AA> = \frac{i}{(M_1 + 4M_3^2 M_4^{-1} \Box)} \Theta_{\mu\nu} + \frac{i}{M_2} \Omega_{\mu\nu}; \quad (3.20)$$

$$<AB> = -2M_3 M_4^{-1} [M_1 + 4M_3^2 M_4^{-1} \Box]^{-1} \epsilon_\alpha^{\lambda\mu\nu} \partial_\lambda \Theta^{\alpha k} \quad (3.21)$$

$$<BB> = \frac{i}{M_4 M_1^{-1}(M_1 + 4M_3^2 M_4^{-1} \Box)} (P_b^1)_{\alpha\beta\gamma k} + \frac{i}{(4M_3^{-1} M_1^{-1} \Box - M_5)} (P_e^1)_{\alpha\beta\gamma k}. \quad (3.22)$$

As for the $\Sigma - \Sigma$ propagator, it comes out as

$$<\Sigma\Sigma> = -[\alpha\zeta\eta^2(\frac{1}{\alpha}\Box - 2h^2\zeta\eta^2)]^{-1} = [2\alpha\zeta\eta^2 M_2]^{-1} \quad (3.23)$$

The poles of the transversal part of the A_μ- and $B_{\mu\nu}$- propagators (3.20)-(3.22) are the same and given by

$$k^2 = 2h^2\zeta\eta^2 + 8\frac{(m + hg\zeta\eta^2)^2}{(1 - g^2\zeta\eta^2)}. \quad (3.24)$$

Therefore, A_μ and $B_{\mu\nu}$ no longer mediate long- range interactions: their physical degrees of freedom combine into physical massive quanta responsible for the short-range character of the interaction. Next, we shall use these results to infer about a vortex-like configuration leading to a particular type of cosmic string.

4 The Cosmic-String Field Configuration

We will finally analyse the possibility of obtaining the cosmic string based on the bosonic Lagrangian eq.(2.16).

The vacuum of the theory is obtained by analysing the minimum of the potential given by eq.(2.14). This system has an extremum at $|\phi| = 0$. This extremum corresponds to an unstable one and, of course, does not provide cosmic string formation. The minimum is set by the equation $\rho = \frac{h\eta^2\zeta}{2m}$.

The minimization of our potential reveals an interesting feature: in the string core, neither gauge symmetry nor supersymmetry are broken; on the other hand, outside the defect, gauge symmetry is spontaneously broken by $<\phi>$, while su.sy. is still kept exact.

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In the string core: $< \phi >=< M >= 0$, so no breaking takes place. Outside the string core: $< \phi >= \eta \neq 0$, $< M >= \rho \neq 0$, $U = 0$; only gauge symmetry is broken.

This might be of interest if we keep in mind the possibility that cosmic strings might have been formed before the phase transition leading to su.sy. breaking. This means that su.sy at the string core is an inheritance of the su.sy. era.

This model has a cosmic string solution with vortex configuration,

$$\phi = \varphi(r)e^{i\theta},$$

$$A_\mu = \frac{1}{h}(A(r) - 1)\delta_\mu^\theta. \tag{4.1}$$

This configuration has the same form of ordinary cosmic string [23], parametrized in cylindrical coordinates (t, r, θ, z), where $r \geq 0$ and $0 \leq \theta < 2\pi$. The boundary conditions for the fields φ and $A(r)$ are

$$\begin{array}{ll} \varphi(r) = \eta & r \to \infty \\ \varphi(r) = 0 & r = 0 \end{array} \qquad \begin{array}{ll} A(r) = 0 & r \to \infty; \\ A(r) = 1 & r = 0. \end{array} \tag{4.2}$$

We propose the following vacuum configuration for the M-field:

$$M = M(r), \tag{4.3}$$

with the boundary conditions below:

$$\begin{array}{ll} M(r) = \rho & r \to \infty; \\ M(r) = 0 & r = 0. \end{array} \tag{4.4}$$

The vortex configuration for the dual field, $\tilde{G}_\mu$, which preserves the cylindrical symmetry is written as

$$\tilde{G}_\mu = \frac{G(r)}{gr}\delta_\mu^\theta, \tag{4.5}$$

with boundary conditions given by:

$$\begin{array}{ll} G(r) = 0 & r \to \infty; \\ G(r) = 0 & r = 0. \end{array} \tag{4.6}$$

The analysis carried out for the propagators leads us to propose that the G-field field does not have propagation outside the string. It is sensible to propose that its only non-vanishing component is the angular one (θ−component) , because this is the only one that couples to the gauge field, A_μ.

The Euler-Lagrange equations stemming from eq.(2.16), combined with the vortex conditions by Nielsen-Olesen-[23], when applied to the φ field give us

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$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial\varphi}{\partial r}\right) - h^2\varphi^3 e^{-2gM} + 2hmM\varphi - \frac{\mathcal{H}^2\varphi}{r^2} - 2gM'\varphi' + g^2M'^2\varphi = 0, \qquad (4.7)$$

where the field strenght, $\mathcal{H}$, is defined by

$$\mathcal{H}^2 = A^2 + G^2 + 2AG. \qquad (4.8)$$

The dynamics of the gauge field, A_μ, with the vortex configuration, eq.(4.1), is given by

$$r\frac{\partial}{\partial r}\left(\frac{1}{r}\frac{\partial A}{\partial r}\right) - \frac{2m}{g}hG - 2h^2[A\varphi^2 + G\varphi^2]e^{-2gM} = 0. \qquad (4.9)$$

These equations have a different form from ordinary cosmic string, but are compatible with its vortex configuration.

The dynamics of the M-field is governed by:

$$\frac{1}{r}\mathcal{P}(rM')' - 2g^2\varphi'M'e^{2gM} + g\mathcal{X}^2 - g\frac{\mathcal{H}^2}{r^2}\varphi^2 e^{-2gM} - \mathcal{Y} = 0, \qquad (4.10)$$

where we define

$$\begin{aligned} \mathcal{X}^2 &= g^2\varphi^2 M'^2 + \varphi'^2 e^{2gM}; \\ \mathcal{Y} &= -(hm\varphi^2 + 2ghmM\varphi^2)e^{-2gM} + 2m^2M - gh^2\varphi^4 e^{-4gM}. \end{aligned} \qquad (4.11)$$

The term labeled by $\mathcal{Y}$ comes from the derivative of the potential.

The equation of motion for the tensorial field, $B_{\mu\nu}$, can be written in terms of the dual, $\tilde{G}_\mu$, as

$$[G - \frac{mg}{h}A - g^2\varphi^2\,(A + G)\,e^{-2gM}]' = 0; \qquad (4.12)$$

We point out that the M-and-G-fields are essential in this formulation of ordinary cosmic string. As shown, the M-field appearing in the eq.(2.14) for the potential is responsible for its shape and plays a key rôle for the non-breaking of supersymmetry.

This matter deserves a better discussion. The fact that the potential U is non-negative is a consequence of supersymmetry. However, to be sure that supersymmetry is actually not broken, one must check that U is indeed vanishing for the configuration under consideration. Actually, we can see that, in the string core, by virtue of the boundary conditions (4.2),(4.4) and (4.6), U is zero and so supersymmetry holds true in the interior region. Outside the core, ρ must be so chosen that $U = 0$, if supersymmetry is to be kept exact; namely, ρ and η are related to one another by $2m\rho = h\eta^2 e^{-2g\rho}$. With this relationship, which is a sort of fine-tunning, we can keep supersymmetry also outside the string. So, we conclude that our model is able to accommodate a cosmic string configuration compatible with $N = 1$-supersymmetry. On the other hand, if we are to reproduce the more realistic situation with broken su.sy. outside the string, we simply do not take ρ and η as related by the

fine-tunning we alluded to above. In such a case, we have su.sy. inside the core, as a relic of the string formation in the su.sy era, and no su.sy. outside, in agreement with current models and phenomenology [13, 12].

At this stage, some highlights on the fermionic zero modes are in order. This issue shall be discussed with a great deal of details in a forthcoming paper [24]; however, we may already quote some preliminary results. All we have done previously concerns the bosonic sector of the $N = 1$- CSKR theory; to introduce the fermionic modes which have partnership with the configurations (4.1),(4.3) and (4.5), take advantage from su.sy invariance. By this, we mean that the configurations for the fermionic degrees of freedom may be found out by acting with su.sy. transformations on the bosonic sector. In a paper by Davis et al. [13], this procedure is clearly stated and we follow it here. The su.sy. transformations of the component fields for the $N = 1$-CSKR model may be found in the work of ref.[20]

Going along the steps of ref. [20], we quote below the fermionic configuration, we have worked out:

$$\chi_a = 2i\bar{\varepsilon}^{\dot{a}} e^{i\theta} \left[\sigma^1_{a\dot{a}} \varphi' + i\sigma^2_{a\dot{a}} A\varphi \right], \tag{4.13}$$

$$\xi_a = 2\bar{\varepsilon}^{\dot{a}} \left[\sigma^1_{a\dot{a}} M' - \frac{i}{gr} \sigma^2_{a\dot{a}} G \right], \tag{4.14}$$

$$\lambda_a = \varepsilon_a \left[2mM - \frac{1}{2} h\varphi^2 e^{-2gM} \right] - i\varepsilon^b \sigma^1_{b\dot{a}} \sigma^{2\dot{a}}_a A', \tag{4.15}$$

where $\sigma^{1,2}$ refer to $r, theta$-components in cylindrical coordinates.

It is worthwhile to notice that the su.sy. transformations lead to a vector supermultiplet ($\mathcal{V}$) that is no longer in the Wess-Zumino gauge; to reset such a gauge for $\mathcal{V}$, we have to supplement the su.sy transformation by a suitable gauge transformation that has to act upon the matter fields as well. $\mathcal{G}$ is gauge invariante, so this further gauge transformation should be carried out only on Φ and $\mathcal{V}$; the results presented in eqs.(4.13)- (4.15) take already into account the c

ombined action of a su.sy. and a gauge transformation.

The equations of motion above do not have an exact solution; they can may be better handled by considering the Bogomol'nyi limit [25]. We show that in the supersymmetric CSKR theory, we can find the limit where the equations of motion can be written as first order equations. In the next section, we will show that the M-and-G-fields may be related to each other by means of the Bogomol'nyi equations.

5   Bogomol'nyi limit

In this section, we show that it is possible to find the energy-momentum tensor for a thin cosmic string in the CSKR theory, in the flat space and we analyse the possibility of to find a Bogomol'nyi solution compatible with the Bogomol'nyi bound. The energy-momentum tensor is defined as

$$T_\nu^\mu = 2g^{\mu\alpha}\frac{\partial\mathcal{L}}{\partial g^{\alpha\nu}} - \delta_\nu^\mu\mathcal{L}. \tag{5.1}$$

The non-vanishing components of the energy-momentum tensor are

$$T_t^t = T_z^z = \varphi'^2 e^{-2gM} + \frac{A^2\varphi^2}{r^2}e^{-2gM} + \mathcal{P}M'^2 + \left(\frac{A'}{rh}\right)^2 + \mathcal{R}\left(\frac{G}{gr}\right)^2 + U. \tag{5.2}$$

From (5.2) we see that the boost symmetry is not break. This is related to the absence of current in z-direction, as in the usual ordinary cosmic string.

The transverse components of the energy-momentum tensor are given by:

$$T_r^r = -\varphi'^2 e^{-2gM} + \frac{A^2\varphi^2}{r^2}e^{-2gM} - \mathcal{P}M'^2 - \left(\frac{A'}{rh}\right)^2 + \mathcal{R}\left(\frac{G}{gr}\right)^2 + U; \tag{5.3}$$

$$T_\theta^\theta = \varphi'^2 e^{-2gM} - \frac{A^2\varphi^2}{r^2}e^{-2gM} + \mathcal{P}M'^2 - \left(\frac{A'}{rh}\right)^2 - \mathcal{R}\left(\frac{G}{gr}\right)^2 + U, \tag{5.4}$$

where $\mathcal{P}$ is the same that was defined in eq.(2.17), and $\mathcal{R}$ is

$$\mathcal{R}(r) = 1 + g^2\varphi^2 e^{-2gM}. \tag{5.5}$$

This energy-momentum tensor has only an r-dependence.

We define the energy density per unit of length, $\mathcal{E}$

$$\mathcal{E} = 2\pi\int_0^\infty T_t^t r\, dr, \tag{5.6}$$

with T_t^t given by Eq.(5.2). This can be written as

$$\mathcal{E} = \ 2\pi\int r\, dr\left[\tfrac{1}{2}\left(\tfrac{A'}{rh} - h\varphi^2 e^{-2gM} + 2mM\right)^2 + \tfrac{1}{r}\varphi^2 e^{-2mM}A' - \tfrac{2}{rh}mMA' + \right.$$
$$\left. + \left(\varphi' - \tfrac{A\varphi}{r}\right)^2 e^{-2gM} + \tfrac{2}{r}\varphi\varphi'Ae^{-2gM} + \left(M' + \tfrac{G}{gr}\right)^2 - \tfrac{2}{gr}GM'\right]; \tag{5.7}$$

and the energy is given by

$$\mathcal{E} = 2\pi\int r\, dr\left[\frac{1}{r}\left(\varphi^2 A\right)' e^{-2gM} - \frac{2m}{rh}A'M - \frac{2}{r}GM'\right]. \tag{5.8}$$

Integrating by parts, and using the boundary conditions, we find

$$\mathcal{E} \geq 8\pi m\rho/h. \tag{5.9}$$

In this Bogomol'nyi limit, there is not forces between vortices and the equations of motion are of first order. The search for a Bogomol'nyi bound for the energy yields the following system of equations:

$$\varphi' - \frac{A\varphi}{r} = 0$$
$$\frac{A'}{rh} - h\varphi^2 e^{-2gM} + 2mM \qquad (5.10)$$
$$M' + \frac{G}{gr} = 0,$$

with G given by eq.(4.9). With these equations, the transversal components of the energy-momentum tensor density vanish and we get in the sense of the distribution:

$$T^\mu_\nu = \mathcal{E} \, diag(1, 0, 0, 1)\delta(x)\delta(y) \qquad (5.11)$$

This analysis provides a complete description of bosonic cosmic strings and we realize that is possible to find a consistent energy-momentum tensor on the bosonic sector.

6 Conclusions

In this work, we have shown that we may obtain a cosmic string in a supersymmetric scenario with the peculiarity that supersymmetry breaking may not take place. The fact that susy might (under certain conditions) be kept exact in the CSKR theory is mainly due to the presence of the M-field, partner of the $B_{\mu\nu}$-field, introduced in the Kalb-Ramond superfield.

Introducing gravity in the model opens up a series of interesting questions to tackle. Gravity by itself is a relevant matter in connection with cosmic string. In our model, once we are dealing with supersymmetry, to bring about gravity we must consider the coupling of the action (2.1) to supergravity. $N = 1-$matter coupling to local su.sy. [26, 27, 28, 29] introduces some arbitrary functions into the model; these functions depend on the scalar matter. It would be a pertinent question to try to understand whether (and how) these functions (usually, referred to in the literature as d and g) may be restricted by the condition of string formation. This would allow us to restrict the universe of d- and g functions guided by the condition that cosmic strings form up in the presence of gravity.

Another point that we would like to address to concerns the topological charge of our solution. ¿From the analysis of the propagators (Section 3), we clearly see that the correlation functions for the A^μ-and-$B^{\mu\nu}$-fields falls off outside the string. Since the topological charge, Q, gets contribution exclusively from the magnetic scalar, $\mathcal{B}$, (which is the divergence of the vector potential, $\vec{b} \sim B^{0i}$), it readily follows that Q vanishes, for it only feels the Kalb-Ramond potential, $\vec{b}$, at infinity. The stability of the solutions is ensured by the topological charge conservation. We have a solution that lives in the $Q = 0$-sector and it is therefore stable.

Finally, an interesting point we should analyse further concerns the property of superconductivity of the string and the consideration of a non-trivial fermionic background. A non-trivial fermionic background is relevant in connection with supersymmetry breaking. So, by coupling to supergravity, a spontaneous su.sy. breaking. might be induced and a careful analysis of the fermion background would be an interesting issue to be contempleted.

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